

**FEEDBACK, DISK SELF-REGULATION AND GALAXY FORMATION****Joseph Silk**Departments of Astronomy and Physics, and Center for Particle Astrophysics,  
University of California, Berkeley, CA 94720**ABSTRACT**

Self-regulation of star formation in disks is controlled by two dimensionless parameters: the Toomre parameter for gravitational instability and the porosity of the interstellar medium to supernova remnant-heated gas. An interplay between these leads to expressions for the gas fraction and star formation efficiency in disks, and to a possible explanation of the Tully-Fisher relation. I further develop feedback arguments that arise from the impact of massive star formation and death on protogalaxies in order to account for the characteristic luminosity of a galaxy and for early winds from forming spheroids.

**1. Introduction**

Star formation in disks appears to be self-regulated. Cloud aggregation and star formation is controlled by the gravitational instability of a cold disk. The Toomre parameter, which controls the growth rate of gravitational instabilities, is near unity as a function of galactocentric radius, within a critical gas surface density. Gravitational instability drives cloud aggregation and star formation, yet the star formation efficiency in disks is low, allowing the disk gas supply to be long-lived. The porosity of the interstellar medium to supernova remnant-heated gas is significant, and of order unity. Porosity must evidently counter star formation. I argue that there is an anticorrelation between these two dimensionless parameters, the gravitational instability parameter and the porosity, that results in the self-regulation of star formation.

A semi-phenomenological theory exists for star formation in disk galaxies. I show in Section 2 that the gas scale-height is controlled by the porosity of the interstellar medium in such a way that the gas velocity dispersion is constant. Incorporation of the Toomre parameter for gravitational instability allows one to account for the inferred self-regulation of the star formation rate. An interplay between porosity and gravitational instability leads to a tentative explanation of the Tully-Fisher relation, in which there is no explicit dependence on dark halos (Section 3).

Star formation in spheroids has a far less secure foundation in theory and in phenomenology than star formation in disks. Indeed, one could safely say that there is essentially no theory and little in the way of phenomenology. Recourse must be had to relatively crude scaling arguments that center on attempts to account for the origin of the galaxy luminosity function. This constitutes one of the outstanding problems in galaxy formation theory. For example, hierarchical merging of dark matter halos yields too steep a slope for

the resulting luminosity function if mass traces light. However, this is known to be a poor assumption, both from direct measurement of the dependence of mass-to-light ratio  $M/L$  on luminosity  $L$ , as manifested by the fundamental plane, and from theoretical arguments which suggest that dwarf galaxies form stars inefficiently. Feedback from star formation can partially suppress dwarf galaxy formation and thereby flatten the slope of the resulting luminosity function. The apparent increase in the comoving number density of dwarfs with increasing redshift may be a manifestation of such a process.

Accounting for the characteristic luminosity  $L_*$  of bright galaxies presents a more fundamental problem. In hierarchical clustering, there is no limit on the mass accumulated, other than that set by the age of the universe. Yet galaxies are clearly distinct in morphology and luminosity density from galaxy clusters. Bright galaxies have a characteristic luminosity, defined by the Schechter luminosity function,  $L_* \approx 10^{10} h^{-2} L_\odot$ . Explanations of  $L_*$  have hitherto been based on the requirement that baryonic matter must cool within a specified time-scale in order to form stars with even moderate efficiency. However this constraint does not restrict  $L_*$  to lie in the range of galaxy luminosities, for objects of cluster mass are forming at present and the cooling time in rich cluster cores is generally less than a Hubble time.

I review the cooling constraints on galaxy formation (Section 4), and then consider star formation in spheroidal protogalaxies. In Section 5, I develop feedback arguments that arise from the impact of massive star formation and death on protogalaxies. Massive galaxies must form stars efficiently. The protogalactic environment must therefore both be able to cool efficiently to form stars, and yet maintain radiative balance with energy input from dying stars. One can thereby account for the characteristic luminosity of a galaxy. On the other hand, in dwarf and in gas-poor galaxies, the ejecta from supernovae drive galactic winds via the porosity of the volume-dominating hot phase (Section 6). A final section summarizes these various results and their implications.

## 2. Disk Star Formation

The theory of large-scale gravitational instability successfully accounts for many aspects of star formation in the Milky Way and in nearby disks. I will argue that there are two key parameters: the interstellar medium porosity  $P$ , which is a measure of the supernova remnant-heated volume fraction, and the Toomre parameter  $Q$ , which controls disk stability. It is the interplay between  $P$  and  $Q$  that provides the necessary feedback that allows disks to be long-lived, and, I shall hypothesize, self-regulated.

Simple global star formation models can account for many aspects of disk star formation. These include the star formation rate, metallicity and gas surface density as a function of disk radius and age, as well as the metallicity distribution of disk stars (Prantzos and Aubert 1995). These models are based on a semi-phenomenological treatment of disk stability (Wang and Silk 1993), and are generally confirmed by numerical simulations (Steinmetz and Muller 1995). Key observational motivations include the near constancy of the star formation rate over galactic disk age, the proportionality of star formation rate to gas surface density ( $HI + H_2$ ), the fact that the inner regions of star-forming disks are marginally unstable (*i.e.*  $Q \sim 1$ ), and that below a surface density threshold defined by  $Q \gtrsim 1$ , global disk star formation effectively ceases, at least in giant HII regions (Kennicutt 1989). Common to these models is a dependence of star formation rate on differential rotation rate, which controls both the linear growth of gravitational instabilities in the disk and the coalescence rate of molecular clouds (Wyse and Silk 1989).

Define an efficiency of star formation per dynamical time by writing

$$\epsilon = \dot{\rho}_* t_{dyn} / \rho_{gas}, \quad (1)$$

where  $\rho_{gas}$  is the gas density, the star formation rate  $\dot{\rho}_*$  can be written as  $\dot{\rho}_* = r_{SN} m_{SN}$ ,  $r_{SN}$  is the supernova rate per unit volume, and  $m_{SN}$  is the mass in stars formed per Type II supernova. For a solar neighborhood (or Miller-Scalo) initial mass function,  $m_{SN} \approx 250 M_\odot$ . One can write the generic disk star formation rate in the form, from (1), either locally by  $\dot{\rho}_* = \epsilon \rho_{gas} t_{dyn}^{-1}$ , or globally as  $\dot{M}_* = \epsilon M_{gas} t_{dyn}^{-1}$ , (since gas and young stars have similar radial distributions and scale-heights). Disks are observed to have low efficiency at forming stars,  $\epsilon \sim 0.03$ , and this is of course required to maintain the gas supply needed for ongoing star formation.

Now the porosity is given by

$$P = \nu_{SN} \dot{\rho}_* m_{SN}^{-1} = \epsilon \frac{\nu_{SN} \rho_{gas}}{m_{SN} t_{dyn}}, \quad (2)$$

where the SNR 4-volume in the SNR cooling phase is (Cioffi, McKee and Bertschinger 1988)

$$\nu_{SN} = 7.82 \times 10^{12} p_4^{-1.36} n^{-0.11} \zeta^{-0.2} E_{51}^{1.27} \text{pc}^3 \text{yr} \equiv A p_{gas}^{-1.36} \rho_{gas}^{-0.11} \zeta^{-0.2} E_{SN}^{1.27}, \quad (3)$$

$\zeta$  is the gas metallicity relative to the solar value, and the gas pressure  $p_4 \equiv 10^4 (p_{gas}/k) \text{cm}^{-3} \text{K}$ . I take  $p_{gas} = \rho_{gas} \sigma_{gas}^2$ , where  $\sigma_{gas}$  is the gas velocity dispersion. Note that supernovae are a stabilizing influence: as the pressure increases, the porosity is reduced.

I will argue that the star formation efficiency  $\epsilon$  is determined by requiring the porosity  $P$  of the interstellar medium to not be large, and thereby avoid blow-out. I consider the local star formation efficiency  $\epsilon$ , defined by (1). It is plausible to believe that self-regulation must result in maintaining  $P \sim 1$ , since blow-out ( $P \gg 1$ ) reduces the (massive) star formation rate, while  $P \ll 1$  would allow the cold phase to dominate sufficiently that the star formation rate increases. Indeed for our own interstellar medium, observations show that  $P \sim \mathcal{O}(1)$ , although there are contentious arguments about whether  $P = 0.2$  or 1 is closer to what is seen in the solar neighborhood (Shelton and Cox 1994). By adopting the 4-volume (3) swept out by a supernova remnant that terminates its expansion at ambient gas pressure  $p_{gas}$ , I infer from (2) and (3) that

$$\epsilon = (P/A) m_{SN} \zeta^{0.2} E_{SN}^{-1.27} \rho_{gas}^{0.47} \sigma_{gas}^{2.72} t_{dyn}. \quad (4)$$

Now I suppose that interstellar clouds are accelerated by supernova remnants and decelerated by cloud collisions. The clouds must acquire a terminal velocity given by  $\sigma_{gas} = \epsilon \nu_{SN} (t_{coll}/t_{dyn})$ , where the specific momentum injected by supernovae per unit of gas mass that forms stars is defined by

$$v_{SN} = E_{SN} v_c^{-1} m_{SN}^{-1} = 500 E_{51}^{13} n_{gas}^{-\frac{1}{7}} m_{250}^{-1} \zeta^{-3/14} \text{km s}^{-1}.$$

Here  $v_c \approx 413 E_{51}^{\frac{1}{14}} n_{gas}^{\frac{1}{7}} \zeta^{3/14} \text{km s}^{-1}$  is the velocity at which an expanding remnant first undergoes substantial radiative losses. The cloud collision time-scale can be written

$t_{coll} = 2^{1/2}(\mu_{cl}/\mu_{gas})(H/\sigma_{gas})$ . Here,  $\mu_{cl}$  is the cloud column density,  $\mu_{gas}$  is the column density of gas in the galactic disk, and  $H$  is the scale-height of the disk. If the clouds are marginally bound, as seems to be the case in our interstellar medium, and maintained against gravitational collapse by internal pressure support that is approximately equal to the mean interstellar pressure, one has  $\mu_{cl} = (p_{gas}/G)^{1/2}$ . The scale-height  $H = \mu_{gas}/\rho_{gas}$ , and so  $t_{coll} = (6\pi G\rho_{gas})^{-1/2}$ . Combining these expressions, one finds that

$$\sigma_{gas} = 6.90P^{-0.58}n_{gas}^{0.1}E_{51}^{0.2}\zeta^{0.008}\text{km s}^{-1}. \quad (5)$$

Hence porosity self-regulation suffices for the gas to have constant velocity dispersion. As  $P$  increases, momentum transfer is progressively less efficient, and  $\sigma_{gas}$  decreases. Only about 2-3 percent of the injected supernova energy is expended in supplying momentum to the interstellar gas. There is no dependence of disk velocity dispersion on the IMF. *It is remarkable that the gas velocity dispersion is insensitive to all physical parameters, and for a self-regulated ( $P \sim 0.5$ ) disk is close to the observed value, of about  $11\text{ km s}^{-1}$  for the 3-dimensional peculiar velocity dispersion of interstellar molecular clouds within 3 kpc of the sun (Stark and Brand 1989).*

Presumably, the young stars which dominate disk light have the same velocity dispersion and scale-height as the gas. One implication is that the (thin) disk scale-height ( $\equiv \sigma_{gas}^2/G\mu$ ;  $\mu$  is disk surface density) is constant, as observed for the stellar component both in edge-on thin disks as well as for the thick (*i.e.* older) components (de Grijs and van der Kruit 1996), only provided that disk surface density is constant. The disk surface density primarily comes from stars, so I conclude that Freeman's law of constant central surface brightness for luminous spiral galaxy disks is equivalent to the requirement of constant scale-height. Another implication is that low surface brightness galaxies are expected to have thicker disks than normal galaxies. The predicted constancy of disk gas velocity dispersion is likely to be the driver behind both disk scale-height and surface brightness in a more realistic model of disk evolution, because the stars form from the gas, but this issue is beyond the scope of the present discussion.

Next, consider the star formation rate. First I evaluate the star formation efficiency. Now  $t_{dyn} = (2\pi G\rho H/R)^{-1/2}$ , so that

$$\epsilon = (\sigma_{gas}/v_{SN})(\rho_{gas}/\rho)^{1/2}(3/\pi)^{1/2}. \quad (6)$$

Note that  $\epsilon$  decreases with time as the gas fraction decreases. One can write the star formation rate, using (4), as

$$\dot{\rho}_* = \rho_{gas}^{1.74}P^{-0.58}\alpha, \quad (7)$$

where

$$\alpha = A^{0.58}m_{SN}\zeta^{0.22}E_{SN}^{-0.73}v_0^{1.58}(6\pi G)^{0.79} = 10.29m_{250}.$$

In the absence of accretion, one finds the solution  $\rho_{gas} = \rho_i(1 + t/t_*)^{-1.35}$ , where the characteristic star formation time-scale is

$$t_* = (2/3)P^{0.58}\alpha^{-1}\rho_i^{-0.74} = 0.81P^{0.58}n_i^{-0.74}m_{250}^{-1}E_{51}^{0.73}\zeta^{-0.22}\text{Gyr}. \quad (8)$$

The return of mass from evolved stars is easily incorporated as a correction factor into this and other expressions given here for star formation times and rates.

These results have two noteworthy implications. At constant scale-height, the star formation rate per unit disk surface area is proportional to the gas surface density to the 1.74 power, with a dependence on just one parameter: porosity. Not only is this a reasonable fit to data on  $H\alpha$  surface brightness (Kennicutt 1989), but there is a straightforward prediction that at a given gas column density,  $H\alpha$  surface brightness is proportional to  $P^{-0.58}$ . Moreover, rapid star formation is achieved in systems with high initial gas density. This has obvious implications for star formation in early type, bulge-dominated galaxies, where the past star formation rate is inferred to be high (Kennicutt, Tamblyn and Congdon 1994).

I now derive the global star formation rate. Knowledge of the star formation efficiency  $\epsilon$  is the key. Integrating the star formation rate per unit volume (7) over disk volume, and making use of  $R = 2^{1/2}t_{\text{dyn}}v_{\text{rot}}$ , yields

$$\dot{M}_* = 6.74P^{-0.58}v_{\text{rot},200}^3n_{\text{gas}}^{0.24}\left(\frac{\rho_{\text{gas}}/\rho}{0.1}\right)^{3/2}\left(\frac{R/H}{0.1}\right)^{1/2}\text{M}_\odot\text{yr}^{-1}, \quad (9)$$

where  $v_{\text{rot},200} \equiv v_{\text{rot}}/200\text{ km s}^{-1}$ . To proceed further, it is necessary to decide on the physics that controls the disk gas fraction.

In fact, so far, disk self-gravity has not been utilized. The key to determining  $\rho_{\text{gas}}/\rho$  is via consideration of dynamical self-regulation. Define the Toomre parameter by

$$\tilde{Q} = \Omega\sigma_g/(\pi G\mu_{\text{gas}}\beta); \quad Q = \tilde{Q}\beta \quad (10)$$

appropriate for a flat rotation curve, where  $\beta = 1 + (\sigma_g/\sigma_*)(\mu_*\mu_{\text{gas}})$  approximately corrects for the self-gravity of the stellar component (velocity dispersion  $\sigma_*$ , surface density  $\mu_*$ ) and  $\Omega$  is the disk angular velocity. One can also express  $Q$  as  $\mu_{\text{cr}}/\mu_{\text{gas}}$ , where  $\mu_{\text{cr}} \equiv \Omega\sigma_g/\pi G$ . One finds empirically for spiral disks that  $Q \sim 1$  throughout the star-forming region. Using  $Q$  for the moment as an independent variable, one can write the local gas fraction as

$$\rho_{\text{gas}}/\rho = 0.017n_{\text{gas}}^{0.1}P^{-0.58}Q^{-1}v_{\text{rot},200}^{-1}E_{51}^{0.2}\zeta^{0.008}\delta, \quad (11)$$

where  $\delta$  is the ratio of disk to gas scale-heights. If  $P$  and  $Q$  self-regulate with  $P \sim Q \sim 1$ , I have inferred the gas fraction.

One now has

$$\epsilon = 0.07n_{\text{gas}}^{0.19}m_{250}P^{-0.29}Q^{-1/2}v_{\text{rot},200}^{1/2}E_{51}^{-0.83}\zeta^{0.22}\delta^{1/2}. \quad (12)$$

Inserting the expression (11) for the gas fraction into equation (9), one finds that the star formation rate is

$$\dot{M}_* = 1.4v_{\text{rot},200}^{5/2}n_{\text{gas}}^{0.29}m_{250}P^{-0.87}Q^{-3/2}E_{51}^{-0.63}\zeta^{0.22}\delta^{3/2}\text{M}_\odot\text{yr}^{-1}. \quad (13)$$

The inferred star formation rate for the Milky Way Galaxy, with  $P \approx 0.3$ ,  $Q \approx 1$ ,  $\delta \approx 1$  and  $v_{\text{rot}} = 220\text{ km s}^{-1}$ , is about  $5\text{ M}_\odot\text{yr}^{-1}$ , in good agreement with the observed value (*e.g.* McKee 1989; Noh and Scalo 1990). Since the disk becomes more unstable as  $Q$  decreases, the star formation rate must increase and the ensuing massive star formation will drive up the porosity  $P$ , which in turn must have the effect of reducing the cold gas supply and thereby depress the star formation rate. Hence this expression for the star formation rate provides an explicit demonstration of disk self-regulation. Moreover, the self-regulation implies that the associated dispersion in  $\dot{M}_*$  as a function of  $v_{\text{rot}}$  will remain small.

### 3. The Tully-Fisher Relation

Dark matter is irrelevant to the derivation of the star formation rate (13). Gas disk self-gravity includes a contribution from the stars, but dark matter plays a subdominant role in maintaining the rotational velocity in the luminous disk region, as is observed for optical rotation curves (Kent 1988). Even in the outer parts of disks, the stellar component is usually close to its maximum possible value and the shapes of the luminosity profiles and HI rotation curves are correlated, while the relative contributions of the halo and stellar components to the rotation velocity vary significantly with luminosity and/or morphological type (Kent 1987). While low surface brightness and dwarf galaxies are usually dark matter-dominated (*e.g.* Cote, Carignan and Sancisi 1991), even here there are notable counterexamples (*e.g.* Carignan, Sancisi and van Albada 1988).

The preceding result (13) is equivalent to a derivation of the Tully-Fisher relation in the blue band. The blue Tully-Fisher relation is dominated by light associated with current star formation, and the self-regulation of disks ( $Q \sim 1$ ) therefore predicts a slope  $\alpha \approx 2.5$  where  $L \propto v_{rot}^\alpha$ . This slope is close to what is observed in the  $B$  band. The low dispersion in the Tully-Fisher relation may perhaps be understood in terms of  $P$  and  $Q$  self-regulation. Of course, dark matter, and its cosmological evolution, is necessary to establish the actual range of observed rotational velocity and initial gas disk mass. It is the transformation to luminosity that is driven by self-regulation.

In fact, compilations of Tully-Fisher data for available samples (Burstein *et al.* 1995; Strauss and Willick 1995) find that the slope increases systematically with increasing wavelength:  $\alpha = 2.1 - 2.2$  ( $B$ ),  $2.5$  ( $R$ ),  $2.7$  ( $I$ ) and  $4.1$  ( $H$ ). A recent comprehensive  $I$  band analysis of a large sample of galaxies finds  $\alpha = 3.1$  (Giovanelli *et al.* 1997). In the  $I$  band, and especially in the  $H$  band, one is measuring the old stellar populations and therefore needs to include the dominant contribution from stars formed over the entire history of the disk.

The old stellar populations may be responsible for the observed steepening of the Tully-Fisher relation. For example, Dopita and Ryder (1994) find that  $\dot{\mu}_* \propto \mu_I^{0.64}$ , which would result in prediction of steeper  $I$ , and presumably  $H$  if a similar relation extends to longer wavelengths, band slopes relative to the  $B$  band slope. For example, if naively applied to the observed blue slope, this observed correlation would steepen the Tully-Fisher slope from 2.1 to 3.3. The steepening is less if not all of the  $B$  light is associated with current star formation. Hence an explanation of the blue Tully-Fisher relation seems to account for the Tully-Fisher relation at longer wavelengths. This suggests that the concern (Willick 1996) that most of the observed steepening in the  $H$  band may be due to use of aperture magnitudes rather than total magnitudes, as used in the other bands where the entire galaxy is imaged, may not be valid.

Low surface brightness (LSB) galaxies present a challenge to any explanation of the Tully-Fisher relation. These galaxies follow the same Tully-Fisher relation as do normal galaxies, at least in the  $B$  band (Zwaan *et al.* 1995), so that application of the virial theorem implies that if  $L/v_{rot,max}^4 = \text{constant}$ , then  $(M/L)^2 \mu_* = \text{constant}$ . The LSB galaxies, typically a factor 4 lower in central surface brightness than normal galaxies in the sample studied by Zwaan *et al.*, are then inferred to have twice the mass-to-light ratio of normal galaxies, and hence are also a factor of 2 larger in disk scale-length at given  $L$  and  $v_{rot,max}$ , as observed. In practice, the observed  $B$  band Tully-Fisher relation has a slope that differs from 4: if I take the observed slope of, say, 2.2, and apply the virial theorem, I deduce that  $(M/L)^2 \mu_* \propto L^{0.8}$ . At fixed luminosity, the previous conclusion about the increase in  $M/L$  for the LSB galaxies still applies.

However  $Q$  should increase, approximately inversely with  $\mu_{gas}$ , for LSB galaxies. This helps explain why the LSB galaxies form stars per unit surface area at a lower rate per unit mass of gas than do normal galaxies. The characteristic star formation time (8) is long because the initial gas density is low. As disk stability, characterized by  $Q$ , increases, I expect that the star formation rate per unit area and the porosity  $P$  must decrease. Perhaps the gas fraction that forms stars per unit dynamical time is determined by local cloud properties and is constant: one would then infer that  $\epsilon$  is constant and therefore that  $QP^{0.6} = \text{constant}$ . In this case, the Tully-Fisher relation (13) derived above for normal galaxies is identical for LSB galaxies. One does not need to appeal to the dark matter distribution or to the initial specific angular momentum of the protogalactic precursors to resolve the question of why the LSB galaxies satisfy a normal galaxy Tully-Fisher relation. Of course, these other issues must presumably be invoked to explain why LSB galaxies have higher  $M/L$  ratios and/or larger scale-lengths than normal galaxies. However the Tully-Fisher relation is entirely a matter of disk star formation physics, which provides a mechanism for self-regulation of the gas reservoir.

#### 4. Cooling Constraints

I turn now to the question of what determines the characteristic luminosity of a spheroid-dominated galaxy. Cooling is generally considered to be the key to understanding the luminous mass of a galaxy (Rees and Ostriker 1977; Silk 1977). However cooling does not necessarily lead to galaxy formation. Cooling flows in cluster cores are environments where one might expect to see forming galaxies. Only old galaxies are found in cluster cores. Even if cooling flows were to have formed giant cD galaxies in the past, one has to remember that presumed hosts of past, as well as current, cooling flows, namely many clusters and groups, do not contain dominant cD's.

Theoretical arguments converge to a similar conclusion. Specifically, one can straightforwardly show that the mass of gas within a dark matter potential well that can cool within a Hubble time is limited only by the mass of dark matter, and therefore cannot account for the luminous stellar mass. Consider the collapse of gas within a dark halo, represented by an isothermal sphere of cold dark matter that contains gas fraction  $f_{gas}$  with density  $\rho = \sigma^2/2\pi Gr^2$ , constant velocity dispersion  $\sigma$ , and mass  $M(< r) = 2r\sigma^2/G$ . In massive halos,  $T \approx \sigma^2 m_p/3k = 4 \times 10^6 \text{K} (\sigma/300 \text{km s}^{-1})^2$ . The ratio of gas cooling time at radius  $r$  to Hubble time is

$$\frac{t_{cool}}{t_H} = \frac{3nkT}{\Lambda n^2 t_H} = \frac{m_p^2 2\pi Gr^2}{f_{gas} \Lambda t_H} \equiv \left( \frac{r}{r_{c,H}} \right)^2, \quad (14)$$

where the cooling radius

$$r_{c,H} = (f_{gas} \Lambda t_H / 2\pi G)^{1/2} m_p^{-1} = 0.3 (f_{0.1} \Lambda_{24} t_{15})^{1/2} \text{Mpc} \quad (15)$$

and the cooled mass

$$M(< r_{c,H}) = \sigma^2 G^{-3/2} m_p^{-1} (2f_{gas} \Lambda t_H / \pi)^{1/2} = 10^{12} \sigma_{100}^2 (f_{0.1} \Lambda_{24} t_{15})^{1/2} M_{\odot}. \quad (16)$$

Here  $\Lambda_{24} \equiv \Lambda/10^{-24} \text{erg cm}^3 \text{s}$  is the cooling rate,  $t_{15} \equiv t/15 \text{Gyr}$  is the age of the galaxy,  $f_{0.1} \equiv f_{gas}/0.1$ , and  $\sigma_{100} \equiv \sigma/100 \text{km s}^{-1}$ .

Gas cooling within a Hubble time might be relevant to disk galaxy masses, which accumulate by slow infall. One might also expect star formation to occur efficiently within a

dynamical time,  $t_{dyn} = r/\sigma$ , as has been argued for elliptical galaxy formation. In this case,

$$t_{cool}/t_{dyn} = 2\pi r m_p^2 \sigma f_{gas}^{-1} \Lambda^{-1} \equiv r/r_{c,d}, \quad \text{or} \quad r_{c,d} = 0.3 f_{0.1} \Lambda_{24} \sigma_{100}^{-1} \text{Mpc}. \quad (17)$$

The mass that has cooled within a dynamical time is

$$M(< r_{c,d}) = \frac{\Lambda \sigma f_{gas}}{\pi G^2 m_p^2} = 10^{12} f_{0.1} \Lambda_{24} \sigma_{100} M_{\odot}. \quad (18)$$

In the relevant temperature range ( $T \gtrsim 10^7 \text{K}$ ), one can write  $\Lambda \approx \Lambda_{ff} = 2 \times 10^{-27} T^{1/2} \text{erg cm}^3 \text{s}^{-1} \equiv \Lambda_0 \sigma$ , and this expression is appropriate at  $T \gtrsim 10^6 \text{K}$  if the metallicity is very low. In this case,

$$M(< r_{c,H}) = \left( \frac{2 f_{gas} \Lambda_0 t_H}{\pi} \right)^{1/2} \frac{\sigma^{5/2}}{G^{3/2} m_p} = 10^{12} \sigma_{100}^{5/2} f_{0.1} t_{15}^{1/2} M_{\odot} \quad (19)$$

and

$$M(< r_{c,d}) = \left( \frac{\Lambda_0 f_{gas}}{\pi G^2 m_p^2} \right) \sigma^2 = 10^{12} \sigma_{100}^{5/2} f_{0.1} M_{\odot}. \quad (20)$$

Both mass estimates increase without limit as the galaxy halo potential grows, as also found in simulations by Thoul and Weinberg (1995).

Cooling and feed-back constraints have been incorporated into hierarchical galaxy formation using semi-analytic models. However the sharp decline in the galaxy luminosity function above  $L_*$  is not explained. For example, Kauffman, White and Guiderdoni (1993) introduced an arbitrary cut-off to avoid formation of excessively luminous galaxies. Dekel and Silk (1986) demonstrated that feedback helps suppress formation of dwarf galaxies, and later papers incorporated this effect into hierarchical galaxy formation (Lacey and Silk 1991; Lacey *et al.* 1993; Kauffmann, Guiderdoni and White 1994; Cole *et al.* 1994). I now argue that combining the physics of cooling and feedback helps suppress the formation of overly massive galaxies.

## 5. A Derivation of $L_*$

I consider supernova heating and feedback, as a possible means of limiting the mass of cooled gas. Suppose supernovae occur at rate  $R_{SN}$  and each supernova injects  $E_{SN}$  ergs into the interstellar gas, of total mass  $M_{gas}$ . For thermal balance to occur, the gas must be able to radiate away the energy injected. The specific rate of thermal energy radiated is  $\frac{1}{2} \sigma^2 t_{cool}^{-1}$ , and this is therefore set equal to the injected energy rate  $R_{SN} E_{SN} / M_{gas}$ . I will argue below that this situation is stable for massive protogalaxies, and does not lead to a supernova-driven wind. From the generic expression for star formation efficiency (1), I then obtain

$$\sigma^2 = \left( \frac{t_{cool}}{t_{dyn}} \right) 2\epsilon \frac{E_{SN}}{m_{SN}}, \quad \text{or} \quad \sigma = 270 \left( \epsilon_{0.2} E_{51} m_{250}^{-1} \left( \frac{t_{cool}}{t_{dyn}} \right) \right)^{1/2} \text{km s}^{-1}. \quad (21)$$

Note that the star formation efficiency ( $\epsilon \equiv 0.2 \epsilon_{0.2}$ ) is expected to be about 10–20 percent for protoellipticals, as inferred from population synthesis modelling of nearby and distant



galaxies, which requires most of the stars to have formed within 1-2 Gyr (Bruzual and Charlot 1993). A lower efficiency would be difficult to reconcile with starbursts. One would certainly need  $\epsilon > 0.3$  in order to have most star formation underway by  $\sim 30t_{dyn}$ , or  $\sim 3$  Gyr.

The value  $m_{SN} \approx 250M_{\odot}$  is based on scaling to the Milky Way, where the specific SN I rate, which must also be included in momentum injection considerations, is higher than in young galaxies and the star formation rate is  $\sim 5M_{\odot}/\text{yr}$ . However for an IMF enriched in massive stars,  $m_{SN}$  may be substantially smaller. For example, the most extreme possibility considered is that the cluster metallicity, including intracluster gas, is a monitor of the protoelliptical yield, from which one infers that the yield is approximately 4 times higher (in terms of mass of iron per unit mass of stars) than in the Milky Way (Renzini *et al.* 1993). One could achieve this high a yield by lowering  $m_{SN}$  by a corresponding factor, to a first approximation (Elbaz, Arnaud and Vangioni-Flam 1995).

To form bulges and ellipticals, not only must thermal balance be attained, but efficient star formation is required. Population synthesis modelling for both ellipticals and bulges suggests that the characteristic star formation time  $t_*$  is less than a Gyr. Theoretical arguments require the star formation time to not exceed the dynamical time, based on diverse considerations that include dynamical friction settling in major mergers, cloud coalescence and cloud disruption by massive star formation (Silk and Wyse 1997). In order to form stars efficiently, one certainly requires  $t_* < t_{cool} < t_{dyn}$ : this is essential in order to first produce the supernovae that heat the gas, otherwise the gas is too hot to form stars. Hence the conditions to form a galaxy are thermal balance and  $t_{cool} < t_{dyn}$ . The requirement of thermal balance for the protogalactic gas now leads to an upper limit on velocity dispersion:  $\sigma \lesssim \sigma_* \equiv 270(\epsilon_{0.2}E_{51}m_{250}^{-1})^{1/2} \text{ km s}^{-1}$ . The central velocity dispersion of an  $L_*$  elliptical galaxy is approximately  $270 \text{ km s}^{-1}$ , and is obtained as a limiting value if  $\epsilon \approx 0.2$ .

In other words, thermal support of the gas sets a limit on  $\sigma$ , and therefore on galaxy luminosity, since  $L$  is correlated with  $\sigma$  according to the Faber-Jackson relation. It is interesting to note that a byproduct of the supernovae, metallicity, correlates more tightly with  $\sigma$ , and in particular with local  $\sigma$ , than with  $L$  (Fisher, Franx and Illingworth 1995). This suggests that the potential well depth, characterized by  $\sigma$ , is more fundamental to early star formation in ellipticals than the total mass in stars. The explanation for a critical luminosity  $L_* \approx 10^{10}h^{-2}L_{\odot}$  above which the number of galaxies exponentially declines may therefore lie in the upper limit on  $\sigma$  for a protospheroid. Note that spheroids dominate at  $L \geq L_*$ : the galaxies with largest  $\sigma$  (and  $L$ ) are giant ellipticals and early-type (bulge-dominated) disk galaxies.

For disks and ellipticals to have similar potential well depths, one must have  $\epsilon(t_{cool}/t_{dyn}) \approx \text{constant}$ . Supernova momentum input into the interstellar medium has an efficiency of a few percent, and the dissipation time of the gas can be as long as the age of the disk: indeed a long timescale is inevitable from the simple observation that star-forming disks are gas-rich. One may regard the cooling time as a lower bound on the star formation time. Perhaps, if  $t_{cool}$  is interpreted more loosely, one could take the momentum dissipation time-scale as an actual estimate of the star formation time. Hence one would end up with  $\sigma_* \propto (\epsilon t_*/t_{dyn})^{1/2}$ , and therefore similar values for  $\sigma_*$ , both in protoellipticals that form stars with high efficiency over a dynamical time and in protodisks that form stars with low efficiency over many dynamical times. One can make a reasonably strong case for self-regulation to have occurred in disks, and at least in this case actually derive the star formation efficiency (12).

## 6. Protogalactic Winds

I finally show that for low  $\sigma$  galaxies, thermal balance is unattainable and a protogalactic wind is inevitable. More generally, this occurs even in more massive galaxies once the initial gas fraction has dropped below  $\sim 10$  percent. A critical parameter in ascertaining the viability of a galactic wind is the porosity of the interstellar gas that is determined by the network of expanding and interacting supernova remnants. Radiative losses regulate the wind velocity, and it is reasonable to estimate the effective wind velocity as given by the specific momentum injected by supernovae. If the volume fraction  $f$  of hot bubble interiors dominates, so that the porosity  $P \gg 1$  (recall that  $f = 1 - e^{-P}$ ), a wind is inevitable provided that a momentum balance condition is also satisfied (e.g. Doane and Mathews 1993), namely that the momentum per unit mass of gas that is injected into the interstellar medium by supernovae exceeds the escape velocity, or  $v_{SN} > \sigma$ . This leads to a remarkable coincidence, given the estimated value of  $v_{SN}$  : winds can occur (but admittedly do not necessarily occur) in potential wells corresponding to those of depth less than or comparable to  $L_*$  galaxies. A wind is only inevitable in galaxy potential wells with velocity dispersion  $\sigma \lesssim 500 \text{ km s}^{-1}$  provided also that  $P \gg 1$ . I now apply (21) to eliminate  $\epsilon/m_{SN}$  from (2), and use (3) to obtain

$$P = \sigma^{-0.71} \left( \frac{\rho}{\rho_{gas}} \right)^{0.5} \rho_{gas}^{0.04} \left( \frac{A}{2} E_{SN}^{0.27} \zeta^{-0.2} \right) \left( \frac{t_{dyn}}{t_{cool}} \right) (\rho^{0.5} t_{dyn})^{-1}. \quad (22)$$

Note that for a top-heavy initial mass function,  $m_{SN}$  would decrease. However at fixed velocity dispersion  $\sigma$ ,  $\epsilon/m_{SN}$  is constant, so that the porosity  $P$  is independent of the IMF.

Cooling certainly permits star formation if  $t_{cool} > t_{dyn}$  as long as  $t_{cool} < t_0$ , the present age of the universe. This may be the relevant condition in disk galaxies, where the star formation efficiency is at most a few percent. However in order for stars to form much more efficiently, one certainly requires  $t_{cool} < t_{dyn}$ . This condition is probably essential for protoelliptical formation as well as in starbursts. Inserting numerical values for the various constants, I find that

$$P > 0.64 \sigma_{100}^{-0.71} f_{0.1}^{-0.5} n_{gas}^{0.04} E_{51}^{0.27} \zeta^{-0.2} \gamma^{-1},$$

where  $\gamma \equiv (2\pi G \rho)^{0.5} t_{cool} \approx t_{cool}/t_{dyn} \lesssim 1$ . Within the starburst core, the porosity is likely to be large and constant if  $\sigma$  and  $f_{gas}$  are sufficiently small.

Clearly, one cannot avoid high porosity either at low gas velocity dispersion or gas fraction, or in the core. The conditions for a radiatively unstable wind are satisfied. To form stars, cooling is essential. Hence protogalactic winds must undergo radiative cooling and consequently be unsteady. I infer that a wind is inevitable either at low  $\sigma$  or low  $\rho_{gas}/\rho$ , independently of star formation efficiency. At  $\sigma \gtrsim 100 f_{0.1}^{-0.71} \text{ km s}^{-1}$ , a wind is inhibited, since  $P < 1$ . However the earlier discussion requires  $\sigma \lesssim \sigma_*$  in order for the supernova energy input to be radiated, otherwise star formation is suppressed. This helps one understand why luminous galaxies have a relatively narrow range of  $\sigma$ .

## 7. Discussion

Global star formation can be understood via self-regulation. This involves feedback from star formation. In this paper, I have developed feedback arguments that arise from the impact of massive star formation and death on gas-rich galaxies and protogalaxies. Star formation in disks has a more secure foundation in theory and in phenomenology than star formation in spheroids.

The idea underlying Section 2 is that in a self-gravitating gas disk, nonaxisymmetric gravitational instabilities drive cloud coagulation, collapse and star formation. Supernova explosions, as well as HII regions, stir up the interstellar gas, tending to increase the gas velocity dispersion and the gas scale-height. The feedback operates via the overlapping hot interiors of supernova remnants that accelerate swept-up shells of interstellar matter and drive gas out of the disk via interstellar chimneys, ultimately generating a galactic wind if the volume filling factor of the hot gas is sufficiently high. The observed self-regulation of star formation in disks is effectively controlled by two dimensionless parameters: the Toomre gravitational instability parameter  $Q$  and the porosity  $P$  of the interstellar medium to supernova remnant-heated gas. I have argued that  $P$  and  $Q$  act in concert: as  $Q$  decreases,  $P$  increases, with the consequence that the star formation rate, found at specified rotation velocity to reduce to a simple function of  $P$  and  $Q$ , self-regulates. The interplay between  $Q$  and  $P$  leads to an explanation of the blue band Tully-Fisher relation, which is dominated by ongoing star formation.

Of course, the rotation curve, as well as the total mass of the disk, is taken to be specified in this analysis. In fact, the dark matter distribution must account for the rotational velocity, and the primordial baryon fraction in conjunction with initial conditions accounts for the total cooled mass of stars and gas (Navarro and Steinmetz 1997). However the Tully-Fisher relation, and its low dispersion, are due to self-regulation of disk star formation. There is an interesting implication: there is likely to be a broad dispersion in dark mass, and hence also in gas mass, at fixed rotation velocity or luminosity, if disks self-regulate. Cosmological initial conditions indeed imply a broad dispersion in galaxy masses at specified circular velocity (Eisenstein and Loeb 1996). The total gas mass is potentially observable, and since the local gas fraction in the star-forming disk is unchanged, the gas distribution, in the case of additional gas mass, must be more extended.

Star formation efficiency is high in deep potential wells and in gas-rich systems. In particular, the characteristic time-scale for star formation is found to be proportional to the inverse  $3/4$  power of the initial gas density. Given that spheroids have a higher central surface brightness, and therefore density, than disks by about two orders of magnitude, one can understand why early, spheroid-dominated Hubble types form stars more efficiently than later, disk-dominated Hubble types. This is consistent with the observed star formation rates for disks of varying Hubble type (Kennicutt, Tamblyn and Congdon 1994). A complementary argument accounts for the longevity of the gas reservoir against depletion by star formation in low surface brightness galaxies.

The present model of disk star formation is reasonably predictive.  $P$  and  $Q$  should be anticorrelated as a function of galactocentric radius. One can try to measure  $P$  from  $H\alpha$  maps, and  $Q$  is especially sensitive to the rotation curve and surface brightness. Porosity tends to oppose star formation, so that later Hubble types, which form stars at a lower rate per unit disk mass than earlier types, should have higher porosity. Little is known about porosity in star-forming disks, and it may be possible, for example by azimuthally averaging  $H\alpha$  maps or  $HI$  maps and appropriate subtraction of stellar continuum, to quantify measures of the porosity of the hot component. At a given gas surface density,

$H\alpha$  surface brightness should anticorrelate with porosity.

For spheroids, the situation is necessarily less constrained than for disks. One lacks the analog of the theory of disk instability to motivate description of the star formation rate. The only resort is to pure phenomenology. Massive spheroidal galaxies must have formed stars efficiently. The protogalactic environment dissipated thermal energy to form stars, while maintaining radiative balance with energy input from dying stars. If the momentum input from supernovae is not dissipated, or if cooling is ineffective, stars do not form. If it is dissipated too rapidly, more stars form and die until the balance is reestablished. This conjecture helps account for the characteristic luminosity of a galaxy. Specifically, if the three-dimensional velocity dispersion of the spheroid exceeds  $\sim 300\text{km s}^{-1}$ , the baryons do not form stars efficiently. Moreover the reduced cooling efficiency must result in gas heating that, at the very least, maintains the gas velocity dispersion. Over a Hubble time, however, the gas can cool. The implication is that outer halos and galaxy groups contain a reservoir of cold gas clouds. This is by no means inconsistent with inferences from studies of quasar absorption line systems (*e.g.* Steidel, Pettini, Dickinson and Persson 1994; Le Brun, Bergeron and Boisse 1996).

On the other hand, in dwarf and in gas-poor galaxies, the ejecta from supernovae drive galactic winds via the porosity of the volume-dominating hot phase. I brought general arguments to bear on momentum input from supernovae that suggest that the luminosities of the spheroidal components are limited by early protogalactic winds. It is inevitable that the porosity  $P \gg 1$  and an early wind must have been generated. One would expect to find both gas-poor low surface brightness, low luminosity dwarf spheroidals and also gas-rich, star-poor clouds in which star formation has failed to unbind the gas or prevent its later accretion. Low redshift intergalactic Ly alpha clouds are possible manifestations of such objects (Shull, Stocke and Penton 1996).

Armed with a star formation history, one can speculate about enrichment and chemical evolution. Formation of spheroids requires 5-10 times the star formation efficiency that is required to form the disk. Metallicity is a built-in byproduct, since the supernova rate is inferred once the star formation rate is specified. With a normal IMF, there is no problem with metal overproduction even when an early wind is generated. Indeed, the converse applies: there does seem to be a need for enhanced yields from spheroidal systems, as inferred from studies of the intracluster gas in rich clusters of galaxies. The abundance ratios measured for the intracluster gas are characteristic of Type II supernovae, and suggest that metals were prolifically produced in forming spheroids. Most of these metals must have been ejected from the galaxies in early spheroid winds.

Finally, it is tempting to speculate about the origin of the enhanced spheroid yields. For example, the observed yields may be due to a top-heavy IMF in the early phases of spheroid formation, although it certainly is premature to exclude other factors that can conspire to both enhance star formation efficiency and yields. If a nonstandard IMF indeed were present, such early winds can serve the functions of enriching both the intergalactic gas, as sampled via studies of Lyman alpha absorption systems towards high redshift quasars, and the intracluster gas, where spheroidal systems are inferred, via the correlation of iron mass with light, to be the dominant pregalactic enrichment source. Such winds simultaneously allow one to account, at least qualitatively, for various properties of the stellar components of spheroids that were generated at birth (Zepf and Silk 1996). These include enhanced  $Mg/Fe$  ratios seen in luminous ellipticals, the correlation of  $Mg$  abundance with local escape velocity in ellipticals, and even the systematic rise in  $M/L$  with spheroid luminosity as encapsulated in the fundamental plane. Indeed, from the theoretical perspective, such feedback from early spheroid formation is desirable, if not mandatory, in order to prevent

gas overcooling and anomalously small disk formation in hierarchical galaxy formation.

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